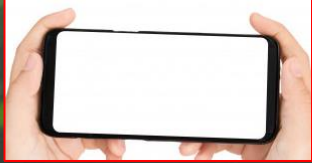
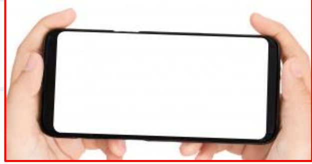
REVISÃO 1º Bim.
Exercícios resolvidos



G.A.A.L.
ENG. DE ALIMENTOS

Prof. Dr. Paulo A. Oliveira

Exercícios Resolvidos



A T E N Ç Ã O :

- Aula presencial
- Ler o livro
- Fazer as listas
- Complementar os estudos

Exercícios Resolvidos

Avaliação Parcial do 1º Bim. - GAAL

Nome: ______________________________ Matrícula: ____________________

Atenção: Não empreste material. Resolver as questões na ordem, com a identificação. Escreva de forma legível. Siga as teorias, os roteiros e as dicas ensinados em aula. Atente-se ao enunciado!

Considere as matrizes e os sistemas abaixo:

$$A = \begin{bmatrix} -3 & -5 \\ 0 & 2 \end{bmatrix},\ B = \begin{bmatrix} 1 & 2 & -2 \\ 4 & 3 & 1 \\ 2 & -1 & 0 \end{bmatrix},\quad C_{2x2} = \{c_{ij} = \mathrm{i}^2 - 2\mathrm{j}\},\quad D_{3x3} : d_{ij} = \begin{cases} 2+i, & i<j \\ 2-j, & i \geq j \end{cases}$$

$$\mathrm{S1} = \begin{cases} 2a - 3b = -4 \\ -2b + 5a = 1 \end{cases} \qquad \mathrm{S2} = \begin{cases} x + 2y - z = -3 \\ -z - 3x = 0 \\ 4y + 2z + 2x = 6 \end{cases}$$

Exercícios Resolvidos

1) Calcule as matrizes C e D.

2) – Resolva as operações matriciais:

a) $F = \frac{1}{2}B + \frac{1}{2}D$ b) $\text{Det}\,(A^t) + \text{Det}(2C)$ c) $H = I_3 + B$

3) Encontre a inversa de $C = -2A$, pelo método das operações elementares;

4) Faça $E = -A^t$ e calcule E^{-1}, pelo método da Ajunta;

5) Resolva S1 pelo método de Cramer;

6) Resolva S2 pelo método Eliminação de Gauss (faça o passo a passo conforme ensinado

7) Encontre os autovalores da matriz C, por meio das contas.

8) Considere H da questão (2.c) e calcule det(H), pela Regra Geral (Teorema de Laplace).

Exercícios Resolvidos

$C_{2x2} = \{c_{ij} = i^2 - 2j\}$, $D_{3x3} : d_{ij} = \begin{cases} 2+i, & i<j \\ 2-j, & i \geq j \end{cases}$

1) Calcule as matrizes C e D.

a) $c_{ij} = i^2 - 2j$

$$C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

$$c_{11} = 1^2 - 2 \cdot 1 = 1 - 2 = -1$$

RESP

$$C = \begin{bmatrix} -1 & -3 \\ 2 & 0 \end{bmatrix}$$

b)

$$D = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix}$$

i) $i < j$ $\quad d_{ij} = 2 + i$

$d_{12} = 2 + 1 = 3$; $d_{13} = 2 + 1 = 3$

$d_{23} = 2 + 2 = 4$

ii) $d_{ij} = 2 - j$

ii) $d_{11} = 2 - 1 = 1$

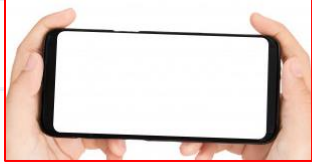
$d_{21} = 2 - 1 = 1$

...

RESP.

$$D = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 0 & 4 \\ 1 & 0 & -1 \end{bmatrix}$$

Exercícios Resolvidos



2) – Resolva as operações matriciais:

a) $F = \frac{1}{2}B + \frac{1}{2}D$ (*) $\quad F = \frac{1}{2}(B+D)$

$B = \begin{bmatrix}1 & 2 & -2\\ 4 & 3 & 1\\ 2 & -1 & 0\end{bmatrix}$,

$D_{3x3} : d_{ij} = \begin{cases}2+i, & i<j\\ 2-j, & i \geq j\end{cases}$

$D = \begin{bmatrix}1 & 3 & 3\\ 1 & 0 & 4\\ 1 & 0 & -1\end{bmatrix}$

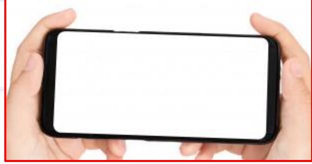
(i) $\frac{1}{2}B = \frac{1}{2}\begin{bmatrix}1 & 2 & -2\\ 4 & 3 & 1\\ 2 & -1 & 0\end{bmatrix} = \begin{bmatrix}1/2 & 1 & -1\\ 2 & 3/2 & 1/2\\ 1 & -1/2 & 0\end{bmatrix}$

(ii) $\frac{1}{2}D = \frac{1}{2}\begin{bmatrix}1 & 3 & 3\\ 1 & 0 & 4\\ 1 & 0 & -1\end{bmatrix} = \begin{bmatrix}1/2 & 3/2 & 3/2\\ 1/2 & 0 & 2\\ 1/2 & 0 & -1/2\end{bmatrix}$

Resolvendo (*)

$F = \begin{bmatrix}1/2 & 1 & -1\\ 2 & 3/2 & 1/2\\ 1 & -1/2 & 0\end{bmatrix} + \begin{bmatrix}1/2 & 3/2 & 3/2\\ 1/2 & 0 & 2\\ 1/2 & 0 & -1/2\end{bmatrix} = \begin{bmatrix}1 & 5/2 & 1/2\\ 5/2 & 3/2 & 5/2\\ 3/2 & -1/2 & -1/2\end{bmatrix}.$

Exercícios Resolvidos



b) $\text{Det}(A^t) + \text{Det}(2C) = k \quad (*)$

$\Rightarrow A = \begin{bmatrix} -3 & -5 \\ 0 & 2 \end{bmatrix}$,

$C_{2x2} = \{c_{ij} = i^2 - 2j\}$

$C = \begin{bmatrix} -1 & -3 \\ 2 & 0 \end{bmatrix}$

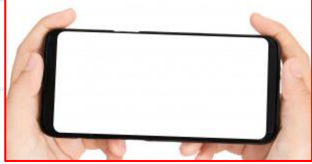
(i) $A^t = \begin{bmatrix} -3 & 0 \\ -5 & 2 \end{bmatrix} \Rightarrow \text{DET}(A^t) = -6 - 0 = -6$

(ii) $2C = 2\begin{bmatrix} -1 & -3 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -2 & -6 \\ 4 & 0 \end{bmatrix} \Rightarrow \text{DET}(2C) = 0 + 24 = 24$

RESOLVENDO (*):

$k = -6 + 24 \Rightarrow k = 18$

Exercícios Resolvidos



c) $H = I_3 + B$

$$B = \begin{bmatrix} 1 & 2 & -2 \\ 4 & 3 & 1 \\ 2 & -1 & 0 \end{bmatrix},$$

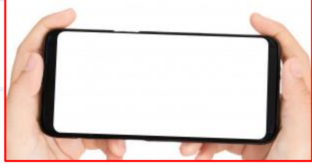
MATRIZ IDENTIDADE

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & -2 \\ 4 & 3 & 1 \\ 2 & -1 & 0 \end{bmatrix}$$

$$H = \begin{bmatrix} 2 & 2 & -2 \\ 4 & 4 & 1 \\ 2 & -1 & 1 \end{bmatrix}$$

Exercícios Resolvidos



3) Encontre a inversa de $\cancel{C}\ P = -2A$, pelo método das operações elementares;

$\cancel{C_{2x2} = \{c_{ij} = i^2 - 2j\}}$

$C = \cancel{\begin{bmatrix} -1 & -3 \\ 2 & 0 \end{bmatrix}}$

$A = \begin{bmatrix} -3 & -5 \\ 0 & 2 \end{bmatrix}$

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

i) $P = -2A \Rightarrow P = -2 \cdot \begin{bmatrix} -3 & -5 \\ 0 & 2 \end{bmatrix}$

$P = \begin{bmatrix} 6 & 10 \\ 0 & -4 \end{bmatrix}$

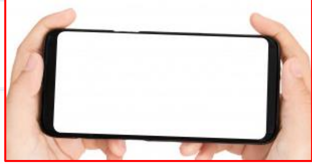
ii) $P \,|\, I_2 = I_2 \,|\, P^{-1}$

$\left[\begin{array}{cc|cc} 6 & 10 & 1 & 0 \\ 0 & -4 & 0 & 1 \end{array}\right] \xrightarrow{L_1 \to \frac{1}{6} L_1} = \left[\begin{array}{cc|cc} 1 & 5/3 & 1/6 & 0 \\ 0 & -4 & 0 & 1 \end{array}\right] =$

$L_2 \to -\frac{1}{4} L_2$

$= \left[\begin{array}{cc|cc} 1 & 5/3 & 1/6 & 0 \\ 0 & 1 & 0 & -1/4 \end{array}\right]$

Exercícios Resolvidos



3) Encontre a inversa de $C = -2A$, pelo método das operações elementares;

$C_{2x2} = \{c_{ij} = i^2 - 2j\}$

$C = \begin{bmatrix} -1 & -3 \\ 2 & 0 \end{bmatrix}$

$L_1 \rightarrow L_1 - \frac{5}{3} L_2$

CONTINUAÇÃO

$\rightarrow \left[\begin{array}{cc|cc} 1 & 5/3 & 1/6 & 0 \\ 0 & 1 & 0 & -1/4 \end{array}\right] \equiv \left[\begin{array}{cc|cc} 1 & 0 & 1/6 & 5/2 \\ 0 & 1 & 0 & -1/4 \end{array}\right]$

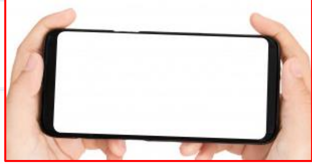
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad I_2 \qquad\qquad P^{-1}$

RASCUNHO

L_1	1	$5/3$	$1/6$	0
$-$ $\frac{5}{3}L_2$	0	$5/3$	0	$-5/12$
	1	0	$1/6$	$5/12$

RESP.: $P^{-1} = \begin{bmatrix} 1/6 & 5/2 \\ 0 & -1/4 \end{bmatrix}$

Exercícios Resolvidos



4) Faça $E = -A^t$ e calcule E^{-1}, pelo método da Adjunta; $A = \begin{bmatrix} -3 & -5 \\ 0 & 2 \end{bmatrix}$.

(i) $E = -A^t = -\begin{bmatrix} -3 & 0 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 5 & -2 \end{bmatrix}$ 0 -6

$A^t = \begin{bmatrix} -3 & 0 \\ -5 & 2 \end{bmatrix}$

(ii) $E^{-1} = \dfrac{1}{DET(E)} \cdot Adj(E)$ (*)

$E \cdot E^{-1} = I_2$

$c_{ij} = (-1)^{i+j} \cdot |C_{ij}|$

$c_{11} = (-1)^2 \cdot |-2| = -2$

$c_{12} = (-1)^3 \cdot |5| = -5$

$DET(E) = -6+0 = -6$

$Adj(E) = cof(E)^t$

$cof(E) = \begin{bmatrix} -2 & -5 \\ 0 & 3 \end{bmatrix} \Rightarrow Adj(E) = \begin{bmatrix} -2 & 0 \\ -5 & 3 \end{bmatrix}$

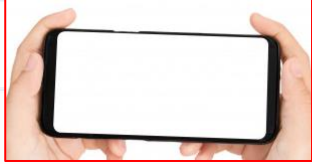
(*) $E^{-1} = \dfrac{1}{-6} \begin{bmatrix} -2 & 0 \\ -5 & 3 \end{bmatrix} \Rightarrow E^{-1} = \begin{bmatrix} 1/3 & 0 \\ 5/6 & -1/2 \end{bmatrix}$ RESP

Exercícios Resolvidos

5) Resolva S1 pelo método de Cramer;

$Ax = B$

$$S1 = \begin{cases} 2a - 3b = -4 \\ -2b + 5a = 1 \end{cases}$$



$$S_1 = \begin{cases} 2x_1 - 3x_2 = -4 \\ 5x_1 - 2x_2 = 1 \end{cases}$$

(i) $A = \begin{bmatrix} 2 & -3 \\ 5 & -2 \end{bmatrix}$ $+15$, -4 $\quad B = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$

(ii) $Ax_1 = \begin{bmatrix} -4 & -3 \\ 1 & -2 \end{bmatrix}$ $+3$, 8 $\quad Ax_2 = \begin{bmatrix} 2 & -4 \\ 5 & 1 \end{bmatrix}$ $+20$, 2

(*) $DET(A) \neq 0 \Rightarrow$ S.P.D

(iii) $DET(A) = -4 + 15 = 11$ $\quad DET(Ax_1) = 8 + 3 = 11$ $\quad DET(Ax_2) = 2 + 20 = 22$

(iv) $x_1 = \dfrac{DET(Ax_1)}{DET(A)} = \dfrac{11}{11} = 1$; $x_2 = \dfrac{DET(Ax_2)}{DET(A)} = \dfrac{22}{11} = 2$

$S = \{(1, 2)\}$

(*) TESTE

Exercícios Resolvidos

6) Resolva S2 pelo método Eliminação de Gauss

$$S2 = \begin{cases} x + 2y - z = -3 \\ -z - 3x = 0 \\ 4y + 2z + 2x = 6 \end{cases}$$

$$S_2 = \begin{cases} x_1 + 2x_2 - x_3 = -3 \\ -3x_1 + 0 - x_3 = 0 \\ 2x_1 + 4x_2 + 2x_3 = 0 \end{cases}$$

$$A = \begin{bmatrix} 1 & 2 & -1 \\ -3 & 0 & -1 \\ 2 & 4 & 2 \end{bmatrix}$$

+12, 0, +4, +12, 4, 0

DET(A) = 22 ≠ 0

S.P.D

SOL. ÚNICA

ETAPA 0

DET(A) ≠ 0

$a_{11} \neq 0 = 1$

$$A^{(0)} | B^{(0)} = \left[\begin{array}{ccc|c} 1 & 2 & -1 & -3 \\ -3 & 0 & -1 & 0 \\ 2 & 4 & 2 & 6 \end{array}\right]$$

×3

n = 3

2 ETAPAS

LINHA 2

ETAPA 1

PIVÔ = 1

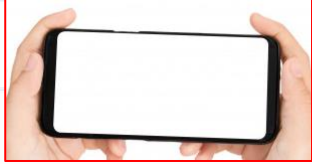
$m_{21} = -3/1 \; ; \; m_{31} = 2/1$

$L_2 \to L_2 + 3/1\, L_1$

$L_3 \to L_3 - 2L_1$

	−3	0	−1	0
+	3	6	−3	−9
	0	6	−4	−9

Exercícios Resolvidos



6) Resolva S2 pelo método Eliminação de Gauss

LINHA 3

$$
\begin{array}{rrrrr}
 & 2 & 4 & 2 & 6 \\
+ & -2 & -4 & 2 & 6 \\
\hline
 & 0 & 0 & 4 & 12
\end{array}
$$

$$
A^{(1)} \mid B^{(1)} = \left[\begin{array}{ccc|c} 1 & 2 & -1 & -3 \\ 0 & 6 & -4 & -9 \\ 0 & 0 & 4 & 12 \end{array}\right]
$$

ETAPA 2 → NÃO É NECESSÁRIA (JÁ ESTÁ PRONTO)

PIVÔ = 6

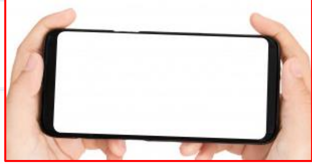
$L_3 \to L_3 + 0 \cdot L_2$

$m_{32} = \frac{0}{6} = 0$

SISTEMA EQUIVALENTE: $\bar{S}_2 = \begin{cases} x + 2y - z = -3 \\ 6y - 4z = -9 \\ 4z = 12 \end{cases}$

TRIANGULAR

Exercícios Resolvidos



6) Resolva S2 pelo método Eliminação de Gauss

L3 $\quad 4z = 12 \Rightarrow \boxed{z = 3}$

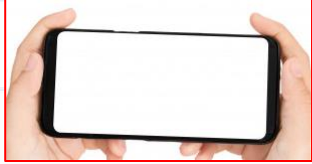
L2 $\quad 6y - 12 = -9 \Rightarrow 6y = -9 + 12 \Rightarrow y = \frac{3}{6} \Rightarrow \boxed{y = \frac{1}{2}}$

L1 $\quad x + 2y - z = -3 \Rightarrow x + 2\left(\frac{1}{2}\right) \cancel{-3} = \cancel{-3} \Rightarrow \boxed{x = -1}$

$$S = \left\{ \left(-1, \frac{1}{2}, 3 \right) \right\}$$

TESTE

Exercícios Resolvidos



7) Encontre os autovalores da matriz C, por meio das contas.

$$C_{2x2} = \{c_{ij} = i^2 - 2j\}$$

$$C = \begin{bmatrix} -1 & -3 \\ 2 & 0 \end{bmatrix}$$

$$C \cdot X = \lambda X$$

$$DET(C - \lambda I) = 0$$

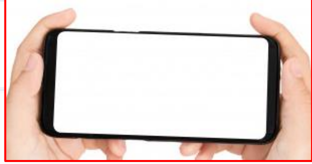
(i) $C - \lambda I_2 = \begin{bmatrix} -1 & -3 \\ 2 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$C - \lambda I_2 = \begin{bmatrix} -1-\lambda & -3 \\ 2 & -\lambda \end{bmatrix}$$

(ii) $DET(C - \lambda I_2) = (-1-\lambda) \cdot -\lambda - 2(-3) = -\lambda - \lambda^2 + 6$

$DET(C - \lambda I_2) = 0 \Rightarrow P_2(\lambda) = -\lambda^2 - \lambda + 6 = 0$

Exercícios Resolvidos



7) Encontre os autovalores da matriz C, por meio das contas.

$C_{2x2} = \{c_{ij} = i^2 - 2j\}$

$$C = \begin{bmatrix} -1 & -3 \\ 2 & 0 \end{bmatrix}$$

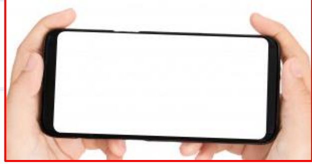
iii) $P_2(\lambda) = 0$ $a = -1$ $b = -1$ $c = 6$

$$\Delta = b^2 - 4.a.c = (-1)^2 - 4.(-1).6 = 25$$

$$\lambda = \frac{-b \pm \sqrt{\Delta}}{2a} \Rightarrow \lambda = \frac{1 \pm \sqrt{25}}{2.(-1)} = \frac{1 \pm 5}{-2} \begin{cases} \lambda_1 = \frac{6}{-2} = -3 \\ \lambda_2 = \frac{-4}{-2} = 2 \end{cases}$$

Resp: os autovalores são $\lambda_1 = -3$ e $\lambda_2 = 2$.

Exercícios Resolvidos



8) Considere H da questão (2.c) e calcule det(H), pela Regra Geral

$n = 3$

$$H = \begin{bmatrix} 2 & 2 & -2 \\ 4 & 4 & 1 \\ 2 & -1 & 1 \end{bmatrix}$$

R.G.

$j = 3$

$$DET(H) = \sum_{i=1}^{3} h_{ij} \cdot (-1)^{i+j} \cdot |\bar{H}_{ij}|$$

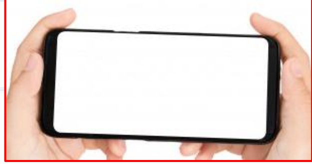
$$DET(H) = \sum_{i=1}^{3} h_{i3} \, (-1)^{i+3} \cdot |\bar{H}_{i3}| \qquad i = 1, 2, 3$$

$$DET(H) = h_{13} \, (-1)^{1+3} \cdot |\bar{H}_{13}| + h_{23} \, (-1)^{2+3} |\bar{H}_{23}| + h_{33} \, (-1)^{3+3} \cdot |\bar{H}_{33}|$$

$$DET(H) = -2 \cdot (-1)^{4} \cdot \begin{vmatrix} 4 & 4 \\ 2 & -1 \end{vmatrix}^{-8}_{-4} + 1 \cdot (-1)^{5} \cdot \begin{vmatrix} 2 & 2 \\ 2 & -1 \end{vmatrix}^{-4}_{-2} + 1 \cdot (-1)^{6} \cdot \begin{vmatrix} 2 & 2 \\ 4 & 4 \end{vmatrix}^{0}$$

$$DET(H) = -2 \, (\underbrace{-8 - 4}_{-12}) - 1 \cdot (-2 - 4)$$

Exercícios Resolvidos



$$DET(A) = -2(-12) - 1(-6)$$

$$DET(A) = 24 + 6$$

$$DET(A) = 30$$